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3 (Sem–2/CBCS) MAT HC 2

2022

MATHEMATICS

(Honours)

Paper : MAT–HC–2026

(Differential Equations)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : ***(any seven)*** $1\times7=7$

(a) Mention *one* principal goal of study of differential equations.

(b) Write down the order of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3+3\frac{dy}{dx}+y=e^x$$

Contd.

(c) What is meant by implicit solution of a differential equation ?

(d) Find the integrating factor of the linear differential equation

$$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$$

(e) State the processes in which compartmental model technique is used to formulate the mathematical model.

(f) Draw the input-output compartmental diagram for CO_2 in earth's atmosphere mentioning the compartment in the model.

(g) Define Wronskian of two functions f and g. What is its value if f and g are linearly dependent ?

(h) The roots of the characteristic equation of a certain differential equation are $0,0,0,3,-4,2\pm 3i$. Write a general solution of the homogeneous differential equation.

(i) Write down the appropriate form of a particular solution of the differential equation

$$y''+3y'+4y=3x+2$$

(j) Find the general solution of the differential equation $y''-9y=0$.

2. Answer the following questions : ***(any four)*** 2×4=8

(a) Determine all values of the constant r for which $y = e^{rx}$ is a solution of $3y'' + 3y' - 4y = 0$.

(b) A function $y = g(x)$ is described by the property that the line tangent to the graph of g at the point (x, y) intersects the x-axis at the point $(x/2, 0)$. Write a differential equation of the form $\frac{dy}{dx} = f(x, y)$ having the function g as its solution.

(c) Find a particular solution of $y'' - 4y = 2e^{3x}$.

(d) State the assumptions made in developing a model of exponential decay and radioactivity.

(e) Reduce the Bernoulli equation

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$$

to linear equation by appropriate transformation.

(f) Verify that $y_1 = 1$ and $y_2 = e^{3x}$ are solutions of the differential equation

$$y'' - 3y' = 0.$$

Also write the general solution of the equation.

(g) Obtain the transformations that can be used to reduce the equation

$$(x - 2y + 1)dx + (4x - 3y - 6)dy = 0$$

to a homogeneous equation.

(h) Determine whether the functions $f(x) = \sin^2 x$ and $g(x) = 1 - \cos 2x$ are linearly independent or linearly dependent on the real line.

3. Answer the following questions : ***(any three)*** 5×3=15

(a) Solve the equation

$$(x^2 - 3y^2)dx + 2xy\,dy = 0$$

(b) Solve the initial value problem

$$\frac{dy}{dx} + y = f(x) \text{ where}$$

$$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & x \ge 1 \end{cases} \qquad y(0) = 0$$

(c) Find a general solution of the differential equation $\frac{dy}{dx} = y^2$.

Find a singular solution of the equation mentioning why is it a singular solution ? 3+2=5 *(d)* State *one* situation where limited growth of population with harvesting model can be useful.

Formulate the differential equation for such model. 2+3=5

(e) Verify that the functions $y_1(x) = e^x$ and $y_2(x) = xe^x$ are solutions of the differential equation $y'' - 2y' + y = 0$. Also find a solution satisfying the initial conditions $y(0) = 3,\ y'(0) = 1$. 3+2=5

(f) The rate at which earth's atmospheric pressure p changes with altitude h above sea level is proportional to p. Suppose that the pressure at sea level is 1,013 milibars and that the pressure at an altitude of 20 *km* is 50 milibars. Use an exponential decay model.

$\frac{dp}{dh} = -kp$ to describe the system, and then solving the equation find an expression for p in terms of h. Determine k and the constant of integration from the initial conditions. What is the atmospheric pressure at an altitude of 50 *km* ? 2+2+1=5

(g) Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' + 3y' + 2y = 4e^x$$

(h) Solve the initial value problem

$y^{(3)} + 3y'' - 10y' = 0$, given

$y(0) = 7,\ y'(0) = 0,\ y''(0) = 70$

4. Answer the following questions : ***(any three)***

10×3=30

(a) Consider the differential equation

$\left(y^2 + 2xy\right)dx - x^2 dy = 0$

(i) Show that this equation is not exact.

(ii) Multiply the given equation through by y^n, where n is an integer and then determine n so that y^n is an integrating factor of the given equation.

(iii) Multiply the given equation through by the integrating factor found in (ii) and solve the resulting exact equation.

(iv) Show that $y = 0$ is a solution of the original non exact equation but is not a solution of the essentially equivalent exact equation.

(v) Name the solution $y=0$ of the given equation. 2+2+4+1+1=10

(b) Suppose the velocity v of a motor boat coasting in water satisfies the differential equation $\frac{dv}{dt}=kv^2$. The initial speed of the motorboat is $v(0)=10m/sec$ and v is decreasing at the rate of 1 m/sec^2 when $v=5$ m/sec. How long does it take for the velocity of the boat to decrease to 1 m/sec ? To $\frac{1}{10}$ m/sec ? When does the boat come to a stop ? 6+2+2=10

(c) Mentioning the assumptions used, formulate the differential equation for exponential growth or decay in population model.
Obtain the solution of the equation. Find an expression for the time for the population to double in size.
5+3+2=10

Solve the initial value problem

$$y''-3y'+2y=3e^{-x}-10\cos 3x$$

Given $y(0)=1$, $y'(0)=2$

(e) Explain the need of density dependent growth model of population over exponential growth model. Formulate the differential equation for density dependent growth model of population.
5+5=10

(f) Find the general solution of

(i) $y^{(4)} = 16y$

(ii) $y^{(3)} + 3y'' + 4y' - 8y = 0$ 4+6=10

(g) (i) Use method of underlined co-efficient to solve $y'' - y' - 2y = 3x + 4$

(ii) Solve the Euler equation

$$x^2y'' + xy' + 9y = 0$$

by appropriate transformations. 5+5=10

(h) The population of a certain city satisfies the logistic law

$$\frac{dx}{dt} = \frac{1}{100}x - \frac{1}{(70)^8}x^2$$

Where time t is measured in years. Given that the population of this city is 1,00,000 in 1980, determine the population as a function of time for $t > 1980$. Hence answer the following :

(i) What will be the population in 2030 ?

(ii) By which year does the 1980 population double ? 6+2+2=10
